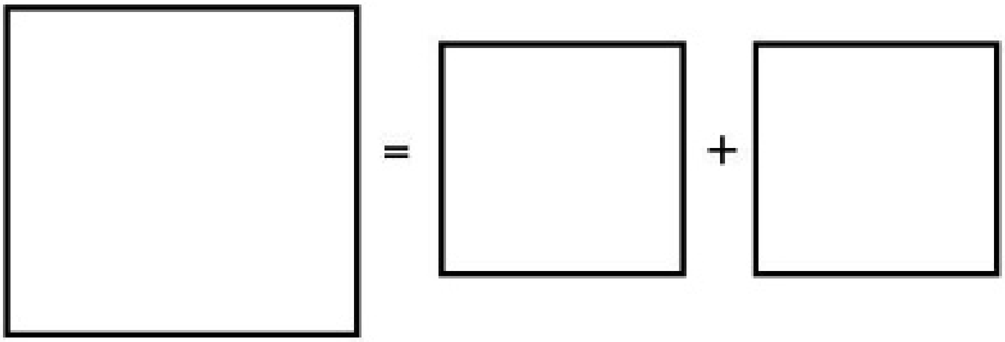
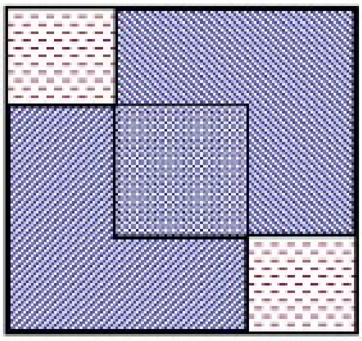
Geometric proof that $\sqrt{2}$ is irrational

The proof that $\sqrt{2}$ is irrational is a few thousand years old, but the proof here dates from around 1990, by some Princeton mathematician. The proof is by contradiction.

$\sqrt{2}$ is rational if and only if there are two (positive) integers, m and n, such that $n^2 = 2m^2$. Equivalently there are two squares, one of side m and one of side n, both integral, such that the second square has twice the area of the first square.



Choose m and n to be the smallest integers for which this holds. Overlay the two smaller squares at the bottom-left and top-right corners of the bigger square.



The overlap of the two smaller squares (the doubly hatched blue area) is another square with an integral side, and this area must be equal to the sum of the two areas of the bigger square which are not overlapped at all (hatched in red), and these two areas are also squares with integral sides. But this contradicts the assumption that m and n are as small as possible.

In algebraic terms, if $n^2 = 2m^2$ then $(2m - n)^2 = 2(n - m)^2$ and $0 < 2m - n < n$.