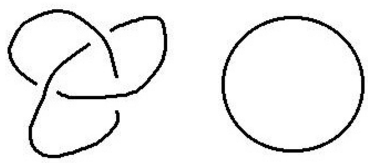
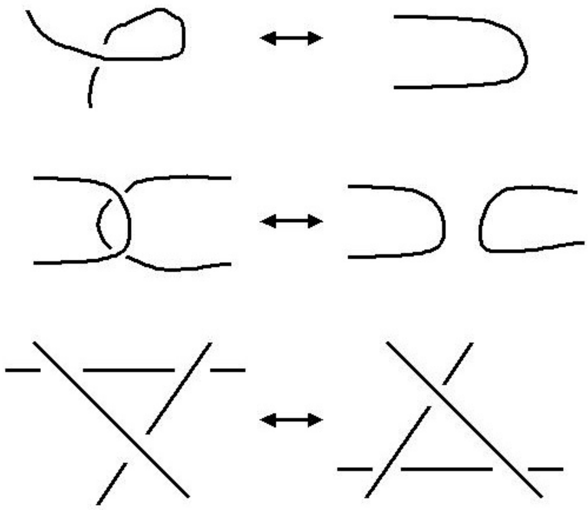
(Part) proof that there is a non-trivial knot

A knot is a map $S^1 \to R^3$. Knots are equivalent if they can be continuously deformed into each other. Knots are usually studied as projections onto R^2. Here we will show that the trefoil knot (on the left) is not equivalent to the trivial knot (on the right).

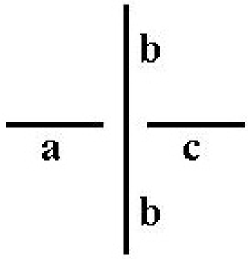


It can be shown that two knots are equivalent if and only if their 2-dimensional projections can be deformed into each other by a sequence of the following (Reidemeister) moves:

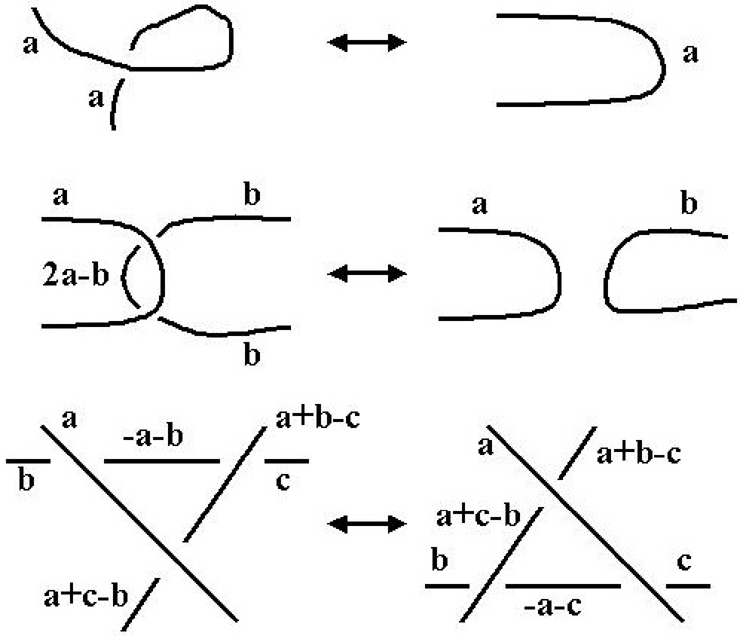


The point of knot theory is to find invariants, i.e. functions on the projection which are invariant under the three moves. If two knots are equivalent the invariant must be the same (but not vice-versa). To prove that the trefoil knot is not equivalent to the trivial knot we will find an invariant which is different for the two.

A numbering of a knot is defined to be a map from the segments of the knot projected in R^2 to $\{0, 1, 2\}$ such that for any crossing:

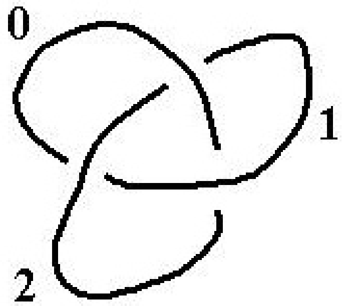


we have $a + c \equiv 2b$ (mod 3). It is easy to see that given any numbering on the left of a Reidemeister move we have a numbering on the right, and vice versa:



The numberings shown are the most general possible, the only tricky one is the last one, where the mod 3 property must be used.

Thus the number of numberings must be an invariant of knots. All knots have at least three numberings, one where all segments are numbered 0, one with 1 and one with 2. These are obviously the only numberings of the trivial knot. But the trefoil knot has another numbering:



Thus it is not equivalent to the trivial knot.